Computer Graphics
CSC 2504 — Assignment 2

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1. (a)

$$\begin{array}{rc} A: & (1,1) \\ B: & (\frac{1}{3},2) \\ C: & (2.5,0.25) \end{array}$$

   (b)

$$M_{BC} = \begin{bmatrix} 0 & -0.5 & 3.5 \\ -1.5 & 0.25 & 0.25 \\ 0 & 0 & 1 \end{bmatrix}$$

2. (a) No.
   Those line families that are normal to the view vector, do not converge to a point and remain parallel. For the given view vector, this means those $B = (x, y, z)$ line families with $3x + 4y = 0$, or $B = (x, -\frac{3}{4}x, z)$.
   For all other line families B which are not normal to the view vector, the family in the VCS has a non-zero z_{vcs} (as $B \times k$ is not zero). So the line would be $A_{vcs} + B_{vcs}t$ which is $(x_a + x_b t, y_a + y_b t, z_a + z_b t)$, all in VCS, and $z_b \neq 0$. Without loss of generality, we assume $z_b < 0$. So the same line after perspective projection would be

$$P = \left(\frac{d(x_a + x_b t)}{(z_a + z_b t)}, \frac{d(y_a + y_b t)}{(z_a + z_b t)}, d \right).$$

   So we get

$$\lim_{t \to \infty} P = (\frac{dx_b}{z_b}, \frac{dy_b}{z_b}, d),$$

   .

   (b)

$$\begin{array}{rcl} k = & \frac{-V_{view}}{|V_{view}|} & = (-0.6, -0.8, 0) \\ i = & \frac{V_{up} \times k}{|V_{up} \times k|} & = (0.8, -0.6, 0) \\ j = & k \times i & = (0, 0, 1) \end{array}$$

   The eye-space transform is:

$$M_{cam}^{-1} = \begin{bmatrix} 0.8 & -0.6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -0.6 & -0.8 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

And projective transform applied to it:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{d} & 0 \end{bmatrix} \begin{bmatrix} 0.8 & -0.6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -0.6 & -0.8 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.8 & -0.6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -0.6 & -0.8 & 0 & 0 \\ \frac{-0.6}{d} & \frac{-0.8}{d} & 0 & 0 \end{bmatrix}$$

(c) $B = (1,0,0)$, so $B_{vcs} = (0.8, 0, -0.6)$. So $z_b \neq 0$ and the line family converges to $(\frac{0.8d}{-0.6}, \frac{0d}{-0.6}, d)$.

3. (a) Let A, B, C, D be the four points of the polygon, which AC is an edge in the given triangulation. We assume that AB, BC, CD, DA are edges of polygon or diameters in the triangulations. Then the algorithm is:
   - If four points A, B, C, D form a convex quadrilateral (in other words: none of them is in the triangle formed by the other three), then return `true`,
   - Otherwise, return `false`.

   (b) Now we can skip the convex check above, so here it is:
   - return `true`,

   (c) Let $P_0, P_1, \ldots, P_{n-1}$ be the vertices of the convex polygon in counterclockwise order, and T_1 and T_2 are the two triangulation of the polygon. Moreover, let T_0 be the unique triangulation of the polygon that P_1 is a vertex of all of the triangles (as the polygon is convex, T_0 always exists).

   As diagonal flips are reversible, all we need to do is to show that both T_1 and T_2 can be turned into T_0 in at most $n-3$ diagonal flips, which the following lemma does. So the proof is complete.

   Lemma For the polygon defined above, and any triangulation T of it, T can be turned into T_0 with at most $n-3$ diagonal flips.

   Proof Let C_n be the number of neede flips to do so. We prove by induction. For $n = 3$, it is trivial, $C_3 = 0$. For $n > 3$, one of the following cases holds:
   - P_1 is a vertex of more than one of the triangles in T: So there should be a diameter in T with P_1 as one of its heads, let P_i be the other head. We can change the two smaller polygons $P_1, P_2, \ldots P_i$ and $P_{i+1}, P_{i+2}, \ldots, P_0$ so that P_1 is a vertex of all of the triangles in them, and we are done. So

     $$C_n \leq C_{i+1} + C_{n-i+1} \leq (i+1-3) + (n-i+1-3) = n-4 \leq n-3$$

   - P_1 is a vertex of only one of the triangles in T: This triangle should be P_0, P_1, P_2. There is another triangle in T which P_0, P_2 is an edge of, let it be P_0, P_i, P_2. We do a diagonal flip on $P_0, P_1, P_i, P2$, and then apply the method of the previous case. So

     $$C_n \leq 1 + C_{i+1} + C_{n-i+1} \leq 1 + (i+1-3) + (n-i+1-3) \leq n-3$$

   The proof is complete now.