CSC2515 — Assignment #2 Answers

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November 9, 2004

1 Adapting Centres in Radial Basis Networks (2%)

- $$\begin{aligned}
\frac{\partial E}{\partial \mathbf{z}_k} &\propto \frac{\partial}{\partial \mathbf{z}_k} \sum_n (y_n - \sum_j w_j h_j(\mathbf{x}_n))^2 \\
&= \sum_n 2(y_n - \sum_j w_j h_j(\mathbf{x}_n)) \frac{\partial}{\partial \mathbf{z}_k} w_k \exp(-\alpha_k (\mathbf{x}_n - \mathbf{z}_k)^\top (\mathbf{x}_n - \mathbf{z}_k)) \\
&= \sum_n 2(y_n - \sum_j w_j h_j(\mathbf{x}_n)) w_k \exp(-\alpha_k (\mathbf{x}_n - \mathbf{z}_k)^\top (\mathbf{x}_n - \mathbf{z}_k)).2\alpha_k (x_n - z_k) \\
&= 4 w_k \alpha_k \sum_n (y_n - \sum_j w_j h_j(\mathbf{x}_n)).h_k(\mathbf{x}_n)(\mathbf{x}_n - \mathbf{z}_k)
\end{aligned}$$

- $$\begin{aligned}
\frac{\partial E}{\partial \log \alpha_k} &\propto \frac{\partial}{\partial \log \alpha_k} \sum_n (y_n - \sum_j w_j h_j(\mathbf{x}_n))^2 \\
&= \sum_n 2(y_n - \sum_j w_j h_j(\mathbf{x}_n))^2 \frac{-\partial}{\partial \log \alpha_k} w_k h_k(\mathbf{x}_n) \\
&= \sum_n 2(y_n - \sum_j w_j h_j(\mathbf{x}_n))^2 \frac{-\partial}{\partial \log \alpha_k} w_k \exp\left(-e^{\log \alpha_k} (\mathbf{x}_n - \mathbf{z}_k)^\top (\mathbf{x}_n - \mathbf{z}_k)\right) \\
&= \sum_n 2(y_n - \sum_j w_j h_j(\mathbf{x}_n))^2 w_k \exp\left(-\alpha_k (\mathbf{x}_n - \mathbf{z}_k)^\top (\mathbf{x}_n - \mathbf{z}_k)\right) .(\mathbf{x}_n - \mathbf{z}_k)^2 \\
&= 2\alpha_k w_k \sum_n (y_n - \sum_j w_j h_j(\mathbf{x}_n)).h_k(\mathbf{x}_n).(\mathbf{x}_n - \mathbf{z}_k)^2
\end{aligned}$$

2 Pseudo-Bayesian Linear Regression (4%)

- We start computing $p(\mathbf{w}|\{\mathbf{x}_n, y_n\}, a, b)$. Since this is a probability distribution, we forget about constants (including the denominator) and finally adjust the solution to sum to one.

$$\begin{aligned} p(\mathbf{w}|\{\mathbf{x}_n, y_n\}, a, b) &= \frac{p(\{\mathbf{x}_n, y_n\}|\mathbf{w})p(\mathbf{w})}{p(\{\mathbf{x}_n, y_n\})} \\ &\propto \prod_n p(\mathbf{x}_n, y_n|\mathbf{w})p(\mathbf{w}) \\ &\propto \exp\left\{\frac{-1}{2b}\sum_n (y_n - \mathbf{w}^\top \mathbf{x}_n)^2 - \frac{1}{2a}\mathbf{w}^2\right\} \\ &\propto \exp\left\{\frac{-1}{2}\left(\mathbf{w}^\top(\frac{\mathbf{I}}{a} + \frac{\sum_n \mathbf{x}_n\mathbf{x}_n^\top}{b})\mathbf{w} - 2\sum_n y_n \mathbf{w}^\top \mathbf{x}_n\right)\right\} \end{aligned}$$

  Now it looks like a gaussian with covariance and mean:

$$\Sigma_{\mathbf{w}} = (\frac{\mathbf{I}}{a} + \frac{\sum_n \mathbf{x}_n\mathbf{x}_n^\top}{b})^{-1}$$

$$\mu_{\mathbf{w}} = \sum_n y_n \mathbf{x}_n \Sigma_{\mathbf{w}}$$

  So:

$$p(\mathbf{w}|\{\mathbf{x}_n, y_n\}, a, b) = \mathcal{N}(\mathbf{w}, \mu_{\mathbf{w}}, \Sigma_{\mathbf{w}}).$$

- We differentiate to find the optimal weights:

$$\frac{\partial}{\partial \mathbf{w}}\cdot = 2\lambda\mathbf{w} - \sum_n 2(y_n - \mathbf{w}^\top \mathbf{x}_n)\mathbf{x}_n$$

3 Regularizing Linear Mixtures of Experts (2%)

- $$\begin{aligned}
\partial \ell^{new}/\partial \mathbf{U}_j &= \sum_n \frac{p(j|\mathbf{x}_n)\partial/\partial \mathbf{U}_j \left[exp(-\frac{1}{2}(\mathbf{y}_n - \mathbf{U}_j\mathbf{x}_n)^\top \Sigma^{-1}(\mathbf{y}_n - \mathbf{U}_j\mathbf{x}_n)\right]}{\sum_j p(j|\mathbf{x}_n)p(\mathbf{y}_n|j,\mathbf{x}_n)} + \partial/\partial \mathbf{U}_j(\lambda \sum_{ijk} U_{ijk}^2) \\
&= \sum_n \frac{p(j|\mathbf{x}_n)p(\mathbf{y}_n|j,\mathbf{x}_n)\Sigma^{-1}(\mathbf{y}_n|\mathbf{U}_j,\mathbf{x}_n)\mathbf{x}_n}{\sum_j p(j|\mathbf{x}_n)p(\mathbf{y}_n|j,\mathbf{x}_n)} + 2\lambda \sum_{ik} U_i jk \frac{\partial}{\partial \mathbf{U}_j}(U_{ijk}) \\
&= \Sigma^{-1} \sum_n p(j|\mathbf{x}_n,\mathbf{y}_n)(\mathbf{y}_n - \mathbf{U}_j\mathbf{x}_n)\mathbf{x}_n + 2\lambda \mathbf{U}_j
\end{aligned}$$

  The last part is deduced by observing that $\frac{\partial}{\partial \mathbf{U}_j} U_{ijk}$ is a matrix of all zeros with a one at entry (i,k).

- To regularize the individual experts, another way is to use ridge regression with each gaussian. Means:

$$\ell = \sum_n \log \sum_j \left[p(j|\mathbf{x}_n)\mathcal{N}(\mathbf{y}_n; \mathbf{U}_j\mathbf{x}_n, \Sigma) + \lambda_j \sum_{ik} U_{ijk}^2 \right]$$

  To regularize the gate, we add $\epsilon \sum v_j^2$ to the cost function:

$$\ell = \sum_n \log \sum_j p(j|\mathbf{x}_n)\mathcal{N}(\mathbf{y}_n; \mathbf{U}_j\mathbf{x}_n, \Sigma) + \epsilon \sum_j v_j^2$$

  To regularize the whole architecture, we add some randomly generated pairs of inputs and output (x_i, y_i), considering the range of each variable when generating them.

4 Fully Observed Trees (10%)

Min log likelihood: -99.57562
Max log likelihood: -5.32291
Mean log likelihood: -14.69724
Median log likelihood: -12.89544
Worst log likelihood belongs to document number 10576
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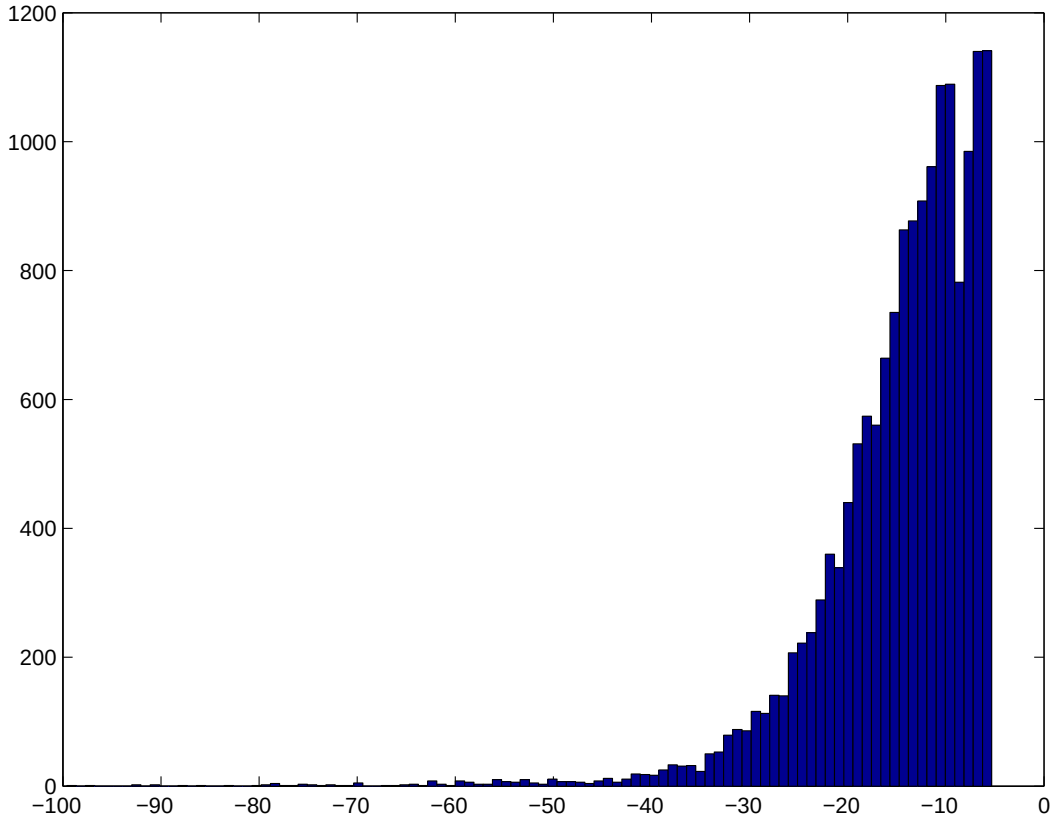

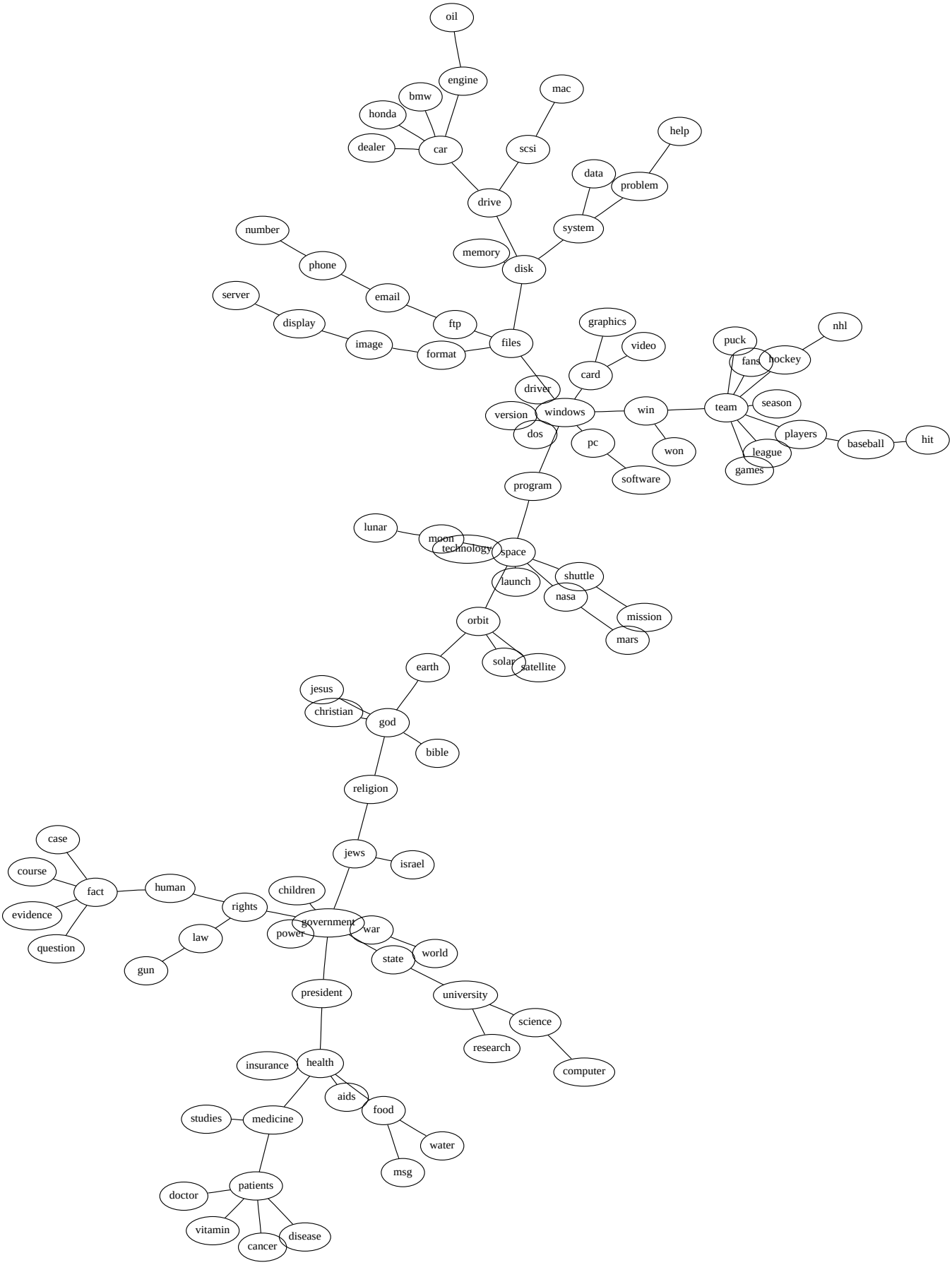
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